Spectral radius of the sum of a random permutation and a deterministic matrix

March 29, 2022

We study the asymptotic behavior of the second eigenvalue of $P := S + Q$ where Q is a given bistochastic matrix and S is a uniformly distributed random permutation of $[N]$. In this goal we consider a sequence of intermediate operators $\tilde{Q} + u$ independent of S which for a given N verifies that $\tilde{Q}$ and Q have same distribution, u is unitary and $(\tilde{Q}, u)$ are free with amalgamation over the diagonal. Under some assumptions over the sparsity of the powers of Q we want to show that $\rho(\tilde{Q}+u)$ bounds asymptotically $\rho(P|_{\mathbb{1}^\perp})$.